Kant's Argument from Incongruent Counterparts Revisited

Abstract
This paper gives an interpretation of Kant's 1768 argument for the reality of absolute space from incongruent counterparts. Although Kant's argument has been subject to sustained attention by Kant scholars and historians of the philosophy of science, there remain interpretive disputes concerning its strategy and whether it successfully establishes its conclusion. I argue that much of this dispute can be resolved by clarifying what Kant means when he claims that space *determines* the objects in it in such a way that they are left- or right-oriented. I show that when Kant claims that space determines the objects in it, he means that left- and right-oriented objects are oriented left and right because although they are exactly similar in size and shape, they occupy different areas that are best understood as different parts of absolute space. In this sense, space determines the objects in it: While most spatial properties do not rely on the existence of absolute space to determine the difference between spatial objects, according to Kant, left and right-handedness could not occur if absolute space did not exist.

Introduction

Look at your right hand, then look at your left hand. Does the left not seem like the mirror image of the right? Now perform an idealization. Assume your hands are perfectly mirrored bodies, despite all their minute differences. They are exactly the same size and shape, but there remains a difference between a left and a right hand. You cannot, for instance, fit your left hand into a right glove no matter how you twist it. Kant calls such objects 'incongruent counterparts.' In 1768, in a paper entitled *On the Ultimate Ground of the Differentiation of Directions in Space*, Kant argued that this ordinary phenomenon proves to us that space is 'absolute': an entity with an independent existence. Kant also considered this phenomenon a decisive counterexample to the relational theory of space, which states that space consists of nothing but the relations among spatial objects (Allais 2015; Hogan 2009a, 2009b; Langton 1996).[1]

Directions in Space drew significant attention as a standalone work, independent of Kant's better-known later arguments for the idea that space and time are mere forms of intuition (Lee and Yelcin 2015; Chalmers 2019; Sklar 1974a; Maudlin 1993).[2] The draw of the paper is patent: It is a short paper written solely to prove the theory of absolute space and it does so by appeal to an ordinary phenomenon. But however captivating the piece is, scholars disagree on even the basic shape of Kant's argument. Not only is it disputed if his argument succeeds, but there is also little agreement on how it establishes its conclusion.

One central point of dispute is the correct interpretation of Kant's argument against the relational theory of space. Kant claims that, on the relational theory of space, a solitary hand in an otherwise empty universe would be neither left nor right. But that, to Kant, is impossible. Any body which has a shape that *can* be oriented left or right *will* be. The dominant view in the literature is that Kant simply has no good argument to sustain this claim (Pooley 2001; Remnant 1963; Gardner 1969; van Cleve 1987).[3] It is perfectly reasonable to suppose that the difference between 'left' and 'right' is a merely semantic difference, but that we have no reason to suppose that objects are left or right in any metaphysically relevant sense. Kant's argument from incongruent counterparts cannot establish any claims about the nature of space, because orientation does not map onto an actual property of objects. But those criticisms of Kant's argument often lack a proper engagement with Kant's ideas on indeterminacy and determination at the time. We don't know why indeterminacy is not an option for Kant and subsequently, we don't understand what, if anything, is responsible for the handedness of bodies on his view. We have found ourselves in trouble: If we don't understand how Kant believes objects in space are determined such that they are handed, we misunderstand what argument Kant is putting forward. I call this interpretive problem 'the indeterminacy problem for the argument from incongruent counterparts'. Regardless of whether Kant scholars come to agree with my metaphysical interpretation of this argument, I believe this paper makes an important and much less controversial contribution in identifying the indeterminacy problem as a central obstacle for interpreting this argument and articulating some constraints on what a good interpretation should look like. My hope is that the scholarship can progress, even if it does so by disagreeing with my view.

In this paper, I argue that Kant's first argument from incongruent counterparts is best understood as establishing that the handed properties of bodies metaphysically depend on absolute space—that is, they are grounded in absolute space. I argue that understanding why things cannot be indeterminately left or right, requires us to understand when things can remain

indeterminate and when they cannot. Further, I show how this enables Kant to state that they require a ground, and that absolute space is the appropriate candidate to ground the handedness of bodies. An upshot of my view is that it coheres more with other developments in Kant's thinking at the time than other interpretations (Beck 1969),[4] and takes him at his word rather than attempting to rescue the argument by redacting what seems opaque. For decades he had concerned himself with the question how to properly and accurately reason about the metaphysics of space without calling into question the scientific consensus. On my view, the argument for the reality of absolute space appears to be a natural consequence of how the discovery of an ordinary fact—the left- and right-handedness of objects—incorporates into a sophisticated theory of metaphysical reasoning.

The paper is structured as follows. In section 1, I lay out the indeterminacy problem in more detail and argue that its solution is necessary to understand the relationship between handedness and theories of space. In section 2, I discuss a proposal to understand the relevant indeterminacy as a problem of mathematical construction, and argue against it. In section 3, I argue for an alternative metaphysical reading for the indeterminacy in question.[5] Having laid this foundation, I then demonstrate in section 4 how absolute space succeeds in grounding a body's handedness and, in this sense, determines it sufficiently.

The Indeterminacy Problem

As I mentioned in my introduction, I believe that much of the disagreement about the shape and success of Kant's argument revolves around how to interpret what Kant means by 'indeterminacy' and under what circumstances it would be impossible for an object to be indeterminately left or right. Since *Directions in Space* has been subject to prolonged analysis, it seems prudent to me to first justify that we have good textual and systematic reasons to worry about indeterminacy. So first, let me present Kant's argument and show that some of the difficulties in understanding the argument necessitate a disambiguation of what Kant means by 'indeterminacy'.

In the central passage where Kant gives the argument from incongruent counterparts, he presents us with a thought experiment. Imagine a single hand were the first thing created in the universe:

> **[a]** '[I]magine that the first created thing was a human hand. That [hand] would have to be either a right hand or a left hand. The action of the creative cause in producing the one would have of necessity to be different from the action of the creative cause in producing the counterpart.
>
> **[b]** Suppose that one were to adopt the concept entertained by many modern philosophers, especially German philosophers, according to which space simply consists in the external relation of the parts of matter which exist alongside each other. It would follow, in the example we have adduced, that all actual space would simply be the space occupied by this hand. **[c]** However, there is no difference in the relation of the parts of the hand to each other, and that is so whether it be a right hand or a left hand; **[d]** it would therefore follow that the hand would be completely indeterminate in respect of such a property. **[e]** In other words, the hand would fit equally well on either side of the human body; but that is impossible.
>
> **[f]** Our considerations, therefore, make it clear that differences and, true differences at that, can be found in the constitution of bodies; these differences relate exclusively to *absolute* and *original space,* for it is only in virtue of absolute and original space that the relation of physical things to each other is possible'. (2:382-3, Kant's emphasis)[6]

Kant presents us here with a fascinating line of reasoning: Assume the only thing existing in the universe were a single human hand, then that hand would have to be either left or right [a]. But suppose the relational theory of space were true and space consisted of nothing but the relations between parts of matter. All space would be only the space occupied by this hand [b]. There is no difference between the parts of matter in the left hand or the right hand [c]. It follows that

the hand would be neither left nor right [d]. But that is impossible [e]. Therefore the differences between incongruent counterparts relate to absolute space [f].

Take a look at the formal features of the argument first. There is an overall argument that concludes in the statement that absolute space is required to make possible the differences between incongruent but similar bodies. Nested in the larger argument for absolute space is a *modus tollens* establishing the falsity of the relational theory of space. If the relational theory of space were true, a hand in an otherwise empty universe would be indeterminately left or right. But that is impossible. Therefore, something over and above the relations between bodies is required to determine left or right-handedness. This argument is valid. But is it sound?

One of the core weaknesses of the argument is the view expressed in [e] that it is impossible for a lone body that has an asymmetrical shape to be neither left nor right but indeterminate in this respect. The claim is central to the success of the argument. If it is not impossible for a hand to be indeterminately left or right, then the argument against the relational theory of space fails, there would be no need to draw a connection between absolute space and incongruent counterparts. Let me call this premise Determinacy:

(Determinacy): If a body is asymmetrically shaped, it will be either left- or right-oriented.

For the argument to succeed, we need to know *why* determinacy is true. Moreso, rejecting determinacy affects other implicit and explicit claims that underwrite Kant's thinking here. These claims concern the question what, if anything, handedness does depend on. Before I discuss how Kant defends his view and what the common criticisms are, I want to draw some little attention to the implications and presuppositions of Determinacy.

Kant's argument implies a general dependence claim, it presupposes that we would reject the idea that handedness results from the internal composition of parts, an implicit assertion that handedness depends on something external to it, and the presupposition that it is absolute space that *can* be that which left-right orientation depends on:

(Dependence): The handedness of bodies depends on something.

This is an implicit assumption in the argument. Kant's argument could not get started without this premise. If we entertained the idea that the handedness of bodies did not depend on anything, we would quickly see that the whole line of reasoning falls apart. Suppose, for instance, that just as bodies are extended and thoughts are not, some bodies that have an asymmetric shape are left-handed and whereas others are right-handed without there being any further reason for it. If he accepted the handedness of bodies as a brute fact, he would not be entitled to make any inferences from incongruent counterparts (Leibniz and Clarke 2000).[7]

(Non-compositionality): The handedness of bodies does not depend on the composition of bodies.

Kant maintains this in [c]: 'there is no difference in the relation of the parts of the hand to each other, and that is so whether it be a right hand or a left hand'. The internal arrangement of hands is insufficient to generate left- or right-handedness. However Kant leaves us without further justification for why that is so.

(Extrinsicality): The handedness of bodies depends on something extrinsic to them.

That handedness depends on something external to them, is quite clearly an implicit assumption, since he concludes that it depends on absolute space.[8] But this claim does not have to be necessarily about space. It can come in various shapes. One could maintain that God created things in this way, or that they are the result of something non-spatial altogether. Perhaps things are left- or right-handed because of other facts about the natural world or facts about the world outside of our universe if such facts exist.

(Non-relationality): The handedness of bodies does not depend on the external relations among bodies.

There are a range of ways that the dependence of handedness on something external may be satisfied. One option is that handedness depends on the relations to other bodies. Kant rejects this idea. The lone hand example asserts that left and right-handedness would still exist if there

were no relations to other bodies in the universe. It is important to note that some spatial properties just are relational in this sense. The relation 'being next to something' is a relation of spatial location that would not occur in the lone hand universe. The lone hand is not next to something because there is nothing else. Kant holds that incongruence is not such a property. A left hand would still be a left hand if the right hand would not exist.

(Absolutism) The handedness of bodies depends on absolute space.

Kant claims this in [e] 'these differences relate exclusively to *absolute* and *original space'*. This is an expression of extrinsicality. Kant here asserts that absolute space is what the differences between incongruent counterparts depend on. The view is that absolute space makes 'the relation of physical things to each other […] possible'. Let us now see if these theses hold up to scrutiny. Although many philosophers seem to agree that the argument has intuitive appeal, the claims I listed here were by no means easily accepted. Peter Remnant, who first drew attention to these claims, argued that Kant's lone-hand thought experiment is plagued by a basic inconsistency. Kant reasons that if the lone hand were indeterminately left or right, it would fit on either side of the human body. This is inconsistent because this argument only works if the human body itself has a shape that is incongruent with its mirror image. Kant is simply affirming the consequent (Remnant 1963; Mühlhöltzer 1992).[9] In other words, Kant is *stating* that it is impossible for the hand to be indeterminate but failing to provide a consistent argument for it (Nerlich 1973).[10] One could be tempted to think that given the historical context, Kant rejected indeterminacy on principle. If this were true, then Kant would have believed in the then popular doctrine of 'complete determination,' in short, the idea that if something *can* be *p*, it must either be *p*, or be *not-p*. And if we wanted to grant him this view, we should see him make use of the doctrine and endorse it somewhere within the argument or prominently elsewhere. But the matter is not straightforward. In the *Critique of Pure Reason,* Kant calls the doctrine of complete determination into question:

> '[…] to know a thing completely, we must know every possible [predicate], and must determine it thereby, either affirmatively or negatively. The complete determination is thus a concept, which, in its totality, can never be exhibited *in concreto'*. (A573/B601)

Kant here expresses the view that there are epistemic constraints on accepting the doctrine of complete determination. Complete knowledge of an object as in complete determination of an object simply surpasses human capacity, so Kant says in his later writings. As I will show later, Kant accepted indeterminacy in certain contexts at the time he wrote *Directions in Space*. But for now, let me hold fixed that this initial route of explaining why Kant endorses Determinacy is not a viable route. We are not in a good position to ascribe to Kant the view that every possible predicate either obtains or does not obtain, without further qualification. The question remains.

The remaining claims in this argument were met with similar scrutiny. John Earman, for instance, took issue with the dependence claim. He argued that handedness is no different from other spatial properties. It is a primitive internal relation, it does not depend on anything, and consequently, 'there is no coherent argument at all'. (Earman 1974: 278) Much like Remnant, Van Cleve, and Pooley, who wondered why objects cannot be indeterminately left or right, Earman wondered why objects have to *depend* on anything for their left- and right-handedness (Remnant 1963: 399).[11] And even if Kant successfully argues *against* the relationality of space, why should we accept that *absolute space* is that in virtue of which things are handed? Things could be handed by an act of God (Earman 1974: 6; Sklar 1974: 278).

The worry arises when considering Kant's phrasing in the sections preceding the one I have cited. Kant states that objects may be perfectly similar, but 'there may remain an inner difference between the two, this difference consisting in the fact, namely that the surface which limits the physical space of the one body cannot serve as a boundary to limit the other, no matter how that surface be twisted and turned, it follows that the difference must be one which rests upon an inner ground' (2:382, my translation). Kant is telling us that left- or right-handedness is an 'inner difference', between objects which they have in virtue of an 'inner ground'. (Hoefer 2000: 239)[12]

And so, for good reason, scholars like Earman have taken Kant to mean that handedness is an intrinsic property, a property that something has by its own nature, and criticized him for it. If things were handed by their own nature, 'it is not at all apparent how introducing an absolute container space will help in distinguishing right from left'. (Earman 1991: 238) This is also a problem for Kant's criticism of the relational theory of space. If bodies were left or right by their own nature, *no* theory of space would account for this. This has led Lawrence Sklar to radicalize Earman's critique. According to Sklar, incongruent counterparts do not tell us *anything* about the nature of space (Sklar 1974: 177ff).

To sum up, what I call the indeterminacy problem and the issues raised by critics of Kant are deeply interconnected. I believe that an answer to why Kant endorsed Determinacy can also hold the key to the various questions of dependence. If we understand better why indeterminacy is not an option, we will gain a more complete picture of Kant's method of reasoning. With this clearer picture in hand, it will become far more transparent which inferences he could draw from determinate properties and the determining agents they depend on.

A Mathematical Solution to the Indeterminacy Problem?

The indeterminacy problem is an interpretive problem: We do not understand well enough why Kant objects to the indeterminacy of a lone hand in an otherwise empty universe. There are two premises in Kant's argument that will be clarified if we disambiguate what Kant means by indeterminacy. First, Kant claims that on the relationist theory of space a handed object would be indeterminately left or right, and second, he claims that objects cannot be indeterminately left or right. The second claim requires an analysis of Kant's views on metaphysical reasoning which he expresses in his writings prior to 1768. Specifically in his 1755 *New Elucidation,* in his 1763 *Negative Magnitudes,* and in his 1765 *New Inquiry,* Kant provides useful context for understanding his thoughts on indeterminacy and dependence. I will provide this analysis in section 3.

But first, why can the relationist not account for incongruence? At first glance, we might be confused: Congruence and incongruence are ways to describe a relation between two objects.

Nothing is *just* incongruent *by itself*, it needs something that it is congruent with. How exactly is incongruence non-relational?

A natural way to put this idea is the following: the scope of an ontological theory of space is to explain what the three-dimensional continuum that contains location and direction of objects consists of. The relational theory of space assumes that spatial objects and an ordering among them (that is, their relations) is all there is. The absolute theory of space stipulates an independent existence of space. If the relational theory of space cannot find a handed property in bodies or reduce incongruence to a relation among bodies, it fails. The most common way to think of one such relation is to find a continuous path between two bodies whereby one body can be superimposed onto the other. Kant defines incongruent counterparts by our ability to enclose a body 'in the same limits' as its mirror image. Such a superimposition is not possible in three-dimensional space. Why does the relationist have to consider the lone hand example? It is, after all, a world without relations to other bodies? Since no external relation or transformation can be found, the relationist is forced to deny that relations between bodies can account for the difference between left and right. The relationist is then asked to consider a universe without such relations and account for the incongruence of objects by their internal relations, by which Kant means the relations between the parts of matter that the lone hand is composed of.

Apart from a few minor exceptions, it has become the consensus view that the relationist cannot find a relation between bodies that can account for the difference between incongruent counterparts. There is a potent interpretation in the literature that sheds light on how to think of the internal composition of objects and this view also provides a reading of what Kant believed was the problem with indeterminacy. According to this reading, Kant's argument targets the geometrical *construction* of incongruent counterparts (Earman 1991; Mühlhölzer 1992; Rusnock and George 1995: 271). On this view, what concerns Kant is the fact that highly abstracted incongruent counterparts such as left- and right-oriented triangles cannot be *constructed* geometrically by using a straightedge and a compass. On this view, an 'internal relation' is a

spatial relation between the parts that constitute an object, as, for instance, Rusnock and George (1995: 261) argue. And if there are no clear means to construct handed objects by using a straightedge and a compass, then such objects are indeterminate in the relevant way.

Although I disagree with this reading, it is important to acknowledge that there are good textual and contextual reasons to endorse it: In his definition of incongruent counterparts, Kant is explicit that moving and turning the object would not allow it to fit into its counterpart's space —'however one twisted and turned [the object]'. (2:382, my translation)[13]. This makes it seem as if Kant is seeking some geometric transformation that would allow us to render objects congruent. The reading receives additional support from the fact that Kant draws connections between his study and geometry. Early in his discussion, he says that his study 'is intended to furnish […] geometers themselves with a convincing argument which they could use to maintain, with the certainty to which they are accustomed, the actuality of their absolute space' (2:387). And within the context of Kant's writings on mathematics, we see him bring up congruence as a mathematical concept. For example, Kant claimed that similarity (i.e. congruence) is well-defined in terms of geometric construction and that philosophical reasoning about its principles is superfluous and inconsequential (2: 277). And lastly, Kant even situates the topic in the tradition of what he calls a certain 'mathematical discipline' (2: 378), referring to Leibniz's *analysis situs* or study of spatial position.

However relevant these remarks look, this interpretation draws a tenuous connection between the idea that internal part-whole relations in objects cannot account for incongruence and a belief in the *completeness* of mathematics. Rusnock and George claim for example that,

> 'Kant had stumbled upon a problem which to his knowledge had not been adequately dealt with by previous geometers, and had a variety of means at his disposal to solve the problem purely mathematically. That he did not do so must be attributed to his belief in the completeness of existing mathematics'. (p. 274)

They claim that Kant believed that mathematics was complete and could not be ameliorated by simply *adding* left- and right-handedness to the list of mathematical primitives:

> '[…]the most promising move would be to include orientation (or "handedness", "the direction towards which the parts are ordered") of figures as a property belonging under the general heading of similarity[…] In received terminology, this would mean adding orientation to the list of *inner* characteristics of figures'. (Rusnock/George 1995 p. 265)

Rusnock and George find Kant's argument to be lacking on precisely these grounds. The problem can simply be solved by adding orientation to the list of mathematical primitives, and so, Kant's argument does not get started.

This reading leaves something to be desired. For one, Kant does not explicitly identify his discussion as a study *in* geometry. Immediately after mentioning Leibniz' *analysis situs*, he clarifies that the task of his own study is to 'philosophically'—that is, metaphysically—determine 'the first ground of possibility' of spatial position. In fact, Kant asserts this twice in the short paper. Kant suggests that geometers should 'make use' of his argument but he does not claim to be doing geometry himself. He also discusses the 'philosophical application of these [mathematical] concepts' (2: 382), and the application of mathematical concepts *in* metaphysics was a topic he had devoted extensive attention to a few years prior to *Directions in Space* (Couturat 1905; Friedman 1985; Hogan 2020).[14]

The fact that Kant extensively investigated the methodological difference between mathematics and metaphysics during the years before the publication of *Directions* points in precisely the opposite direction. Rather than suggesting that Kant believed no geometrical construction could be provided of incongruent counterparts,[15] these writings suggest how careful and intentional Kant was when reasoning metaphysically about space and about the objects of mathematics. The most explicit investigation of this sort is found in the 1764 *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morals*. The paper was written in response to an essay contest in which the *Berlin Academy of Sciences* requested responses to the question whether the metaphysical

truths and geometrical truths were similarly certain and if they were not, what the nature of the certainty of metaphysical truths is. Kant responds to this question by arguing that geometrical truths are certain because of a methodological difference between mathematics and metaphysics. While metaphysics proceeds analytically, mathematics and, geometry more specifically, proceeds synthetically, that is, it constructs its own objects by synthesis:

> 'There are two ways in which one can arrive at a general concept: either by the *arbitrary combination* of concepts, or by *separating out* that cognition which has been rendered distinct by means of analysis'. (2:276)

Kant adds that even though sometimes philosophers offer synthetic explanations, and sometimes mathematicians offer analytic explanations, these explanations are never legitimate. He draws a distinction between the methods of mathematics and the method of metaphysics that focuses on the question of what is required *in explanation* (*Erklärung*).

> 'In mathematics, the definitions are the first thought which I can entertain of the thing defined, for my concept of the object only comes into existence as a result of the definition. It is, therefore, absolutely absurd to regard the definitions as capable of proof'. (2:281)

So, in geometry, certainty is not derived by proof but simply because the definition or explanation of an object brings it into existence. This differs from the metaphysical method which works by analyzing *given* concepts—'the concept of the thing to be defined is given to me'. (2:281)

What Rusnock and George fail to point out is that the consequence of their interpretation is not just that Kant makes a weak argument. The consequence is that Kant would not have an argument from incongruent counterparts at all. If mathematical objects are constructed by synthesis from mathematical primitives, and incongruent counterparts cannot be constructed by such synthesis, then they simply are not mathematical objects by his own standards. If his

concern about the internal relations of bodies was about the geometric construction of objects alone, it is hard to see why Kant would care about them (Kant 2020: 88).[16]

What would a metaphysical investigation into incongruent counterparts in contrast look like? James Van Cleve argues in *Right, Left, and the Fourth Dimension* that this question should concern the proper *grounds* on which something is considered left-handed or right-handed.[17] If the relational theory of space can explain how relations *ground* the orientation of hands, then it is successful (Hogan 2021; Rukgaber 2016).[18]

I build on Van Cleve's work and offer a more natural reading of indeterminacy that does not involve ascribing to Kant the view that mathematics is complete. I want to take Kant's remarks about the metaphysical method seriously and ask what a metaphysical investigation into incongruent counterparts would look like by his own standards. Concepts are *given* to metaphysical inquiry, according to the above definition. If incongruent counterparts are given, what can we know about them by metaphysical reasoning, i.e. analysis? And does this analysis explain what the relationist must account for?

The Metaphysical Approach

The best way to understand Kant's thoughts on what determines spatial properties is to read Kant. Kant discusses various the notion of space, what it grounds and what it can ground in various places before *Direction in Space* was published in 1768. For instance, in his 1755 *New Elucidation*, he argues that space is a set of relational determinations of substances from a series of concerns about how substances ground changes in one another. A year after the publication of the *New Elucidation*, in 1756 Kant writes a paper entitled *The Employment in Natural Philosophy of Metaphysics Combined with Geometry, of which Sample I Contains the Physical Monadology*. In this paper, he provides more insight into how we are meant to understand the idea of spatial properties as relational determinations. The purpose of the *Physical Monadology* is to solve an apparent incompatibility between fundamental insights from geometry and metaphysics: If substances are in space and space as infinitely divisible—as presupposed in geometry—then substances would

be infinitely divisible. But metaphysicians believe in the existence of indivisible substances such as souls. Kant attempts to render these ideas compatible by claiming that substances express themselves *in space* but are not thereby infinitely divisible. More specifically, he argues that bodies consist of indivisible parts of matter, what he calls 'physical monads'. This internal composition of such parts is 'nothing but a relation' (1:477). Those physical monads are expressions of the interaction of substances. How does this work? Substances are endowed with an *active* force that seeks to act on other substances, and since those other substances are likewise endowed with the same force, the result is apparent in their interaction. If only one substance existed and sought to realise its action, it would face no constraints on this expression. But since it is competing with other such substances, and since their interaction is in space, the result is a partial expression of their action and a partial expression of other substances' actions: it is a limitation in space—an extension. Thereby physical monads 'fill a determinate space' (1:481). However, 'in addition to […] the relational determinations of substance, there are other, internal determinations; if the latter did not exist, the former would have no subject in which to inhere. But the internal determinations are not in space, precisely because they are internal' (1:481).

Kant's solution involves arguing that extension is the effect of the action of a substance onto other substances, not a description of its nature.[19] Note how this claim is in important ways distinct from Kant's later claim about what things are like in themselves. Kant believes that we know at least two intrinsic properties of substances: they are indivisible and they are endowed with active and passive forces (Allais 2015: 245-248; Langton 1996: 119; Langton 2006:170-185).[20] He believes that these intrinsic properties express themselves in spatial relations and that taking away the intrinsic properties of substances would result in their external relations being canceled out. But this does not mean that spatial relations are fully reducible to intrinsic properties, because then the relation depends on nothing but their intrinsic properties. Neither are they are completely irreducible. The textual evidence supports the view that relations are

jointly dependent on the intrinsic properties of substances and on something superadded to them.

From these writings alone, we can see that a young Kant endorsed a specific variant of the relational theory of space and that this version of it did not have conceptual resources to account for incongruence. First, the intrinsic properties of substances are not *in space*. Thus, there could be an inner ground of the difference between incongruent counterparts. However, his does not mean that Kant believed that incongruence was an intrinsic property of things. Now, the relational determinations of substances, including extension and the composition of bodies, are a result of the mutual interaction of substance. While those interactions are not fully reducible to substances, neither fully independent of substances, they seem to be strictly defined as a mutual, symmetric form of interaction and any asymmetry would be hard to account for including incongruence. Let me explain.

Place yourself in Kant's universe as described by 1756. We are contemplating some symmetric object in space: a ball. Why is the ball extended? We believe that it is composed of parts of matter, but not *ad infinitum*. There is a finite number of these parts of matter. Those parts of matter fill space. How do we explain that they fill space? We believe that they are the product of substances endowed with forces acting on one another. Their mutual interaction is ensured by God and results in the filling of some determinate space. Note how the construction of such bodies is from parts to whole, i.e. bottom-up. We can explain the extension of the ball because we can explain its composition from parts of matter. We can explain the extension of parts of matter because we can explain how substances interact with each other. This is great! All we wanted to explain is why the ball is extended. It can be explained in this way.

Now consider an asymmetrical object: a hand, a foot, a snail's shell. Attempt to construct it bottom-up. Imagine you arranged some marbles on a table and tried to arrange a snail's shell. You place them next to each other and manage to create it. It works. But it is either left or right

oriented. Can we explain from what we know about space why it is left or right oriented? We have no good explanation available.

Now add to this young Kant's thoughts about space the insight generated later, in 1763 in *Negative Magnitudes.* In *Negative Magnitudes,* Kant considers again what it takes to occupy a space, now within the context of considering what he calls "real grounds." In *Negative Magnitudes,* Kant argues that there is a difference between real grounds and conceptual grounds. A real ground of determination is *really* distinct from the thing, that is, it is distinct from what is conceptually contained in the thing. Kant now adds to this analysis that the origin of "space filling" is based on a *real* ground because it cannot exist unless some *other* substance stands in interaction with that thing.

> 'A body, in virtue of its impenetrability, resists the motive force of another body attempting to penetrate the space which it occupies. In spite of the motive force of the second body, the impenetrability of the first body is nonetheless a ground for that second body's rest. It follows from what has already been said that impenetrability just as much presupposes a true force in the parts of the body, in virtue of which they collectively occupy a space, as does the force in virtue of which another body strives to enter that space'. (2:179)

So, Kant argues that some extensive properties, specifically filling space, have real grounds, because they do not result from the concept of a body but are the result of the interaction of something distinct from them, namely the activity of another body (Stang 2012: 74-101).

The 1764 insight from the *Inquiry* adds to Kant's perspective on this line of reasoning. He argues that metaphysics should restrict itself to analysis from *given* concepts. How would he now account for incongruence? He would start from the question *how* incongruence occurs since incongruence is given in the fact of the matter. Note how his early theory of space is not conceptually potent enough to explain *specific* differences between extended bodies. It explains extension *simpliciter*. Neither intrinsic properties of substances, nor relational determinations, nor

God explain such specific differences. Arguably, the way in which real things are extended is a way in which things occupy space. They would thus need to be the result of a real ground. Kant would be inclined to *amend* his metaphysical concept of space to allow for an explanation either for how the mutual interaction of substance can result in an asymmetric result such as the handedness of bodies, or leave it as such and accept that relations cannot account for left-right determinations.

Now that we have a clearer grasp of what it would require for the relationist—in this case, a younger Kant—to determine handedness, we should address the more general question of *why* indeterminacy is not an option for Kant.

Recall that there were two questions that required our attention: (i.) Why would Kant think that the relationist cannot account for the idea that orientable objects can remain indeterminate and (ii.) Why does he rule out indeterminacy? In answer to the first question, I just now suggested that we can see an earlier relationist theory in Kant's own writings that lacked the conceptual resources to explain asymmetric spatial extension. But given Kant's investment in metaphysical reasoning by analysis rather than synthesis, he found himself in a position to amend his previous theory of space to account for incongruent counterparts. What is my response to the second question?

Recall that Kant critically discusses the principle of complete determination in his mature philosophy. During the time when Kant was writing *Directions*, the question of indeterminacy was discussed as a *specific* problem that occurs when we think about *objects*, and it is not a problem when we think about concepts. Nick Stang argues convincingly that in *The Only Possible Ground*, Kant believed that 'objects are fully determinate but concepts of them can be incompletely determinate'. (Stang 2016: 63)

Is Kant worried about objects or concepts here? As I stated at the outset, Kant believed that hands are determinately left or right. And he also stated that we have an actual experience of such a property in our hands. This explains why he rules out the left-right indeterminacy of

objects that can be handed. He would not worry about the concept of a hand not containing the left/right orientation, but an actual hand is either left or right.

What we further have evidence for is the idea that a determinate property requires a ground and so enables metaphysical analysis. In his 1755 *New Elucidation of the First Principles of Metaphysical Cognition,* Kant gives the fullest characterization of his notion of a ground within the context of articulating a principle of cognition.[21] The principle of determining ground now states that everything has a determining ground. Kant uses a very general definition of ground here:

> 'That which determines a subject in respect of any of its predicates, is called the ground' (1:391).

Kant goes on to differentiate kinds of grounds:

> 'Grounds may be differentiated into those which are antecedently determining and those which are consequentially determining. An antecedently determining ground is one, the concept of which precedes that which is determined. That is to say, an antecedently determining ground is one, in the absence of which that which is determined would not be intelligible. A consequentially determining ground is one which would not be posited unless the concept which is determined by it had not already been posited from some other source' (1:391 f.).

Kant here defines two types of grounds: antecedently determining grounds and consequentially determining grounds. Antecedently determining grounds explain why the existence of something or its features are intelligible, because were those grounds absent, the thing itself would make no sense. For example, the antecedently determining ground of a paper being inky would be someone moving their pen on the paper. This is meant to make the consequence intelligible. If no one were moving their pen on paper, it would be strange for the paper to be inky. It is not intelligible why the paper is now suddenly inky. So far, so good. What is a consequentially determining ground? A consequentially determining ground is somehow derivative. Kant provides an example: observations of Jupiter's satellites gave scientists evidence that light was

propagated with a measurable velocity, but if we took the satellites away, the velocity of light would not change. A ground must be distinct from the thing it grounds and it can be distinct in various ways. If I am in search of logical ground, I can find it by considering the concept of a thing, and finding distinct characteristics in its concept that enabled me to determine its properties. And if I am in search of a real ground, the ground must be *really* distinct from the thing it grounds.

This now helps us better understand that when Kant is searching for a ground of the difference between handed objects, he is searching for either something that is contained in the concept of the thing or for something else really distinct from it. Is Kant indicating what kind of ground he is looking for in this paper? Conceptually, incongruent counterparts are the same:

> 'The right hand is similar and equal to the left hand. And if one looks at one of them on its own, examining the proportion and the position of its parts to one another, and scrutinizing the magnitude of the whole, then a complete description of the one must apply in all respects to the other as well' (2:381).

So, their difference must have a real ground in something not contained in their description alone but quite simply in some *real* entity.

The Case for the Reality of Absolute Space

We are now in a better position to reassess the argumentative strategy of *Directions* and make the case for the reality of absolute space. Recall from the last section that the ground of incongruent counterparts must determine the difference between left and right and that it must be a real ground.

To argue the case for how absolute space features in the argument, all I want to argue for right now is that absolute space is the best candidate to be a real ground, not that it rules out other views. I argue that Kant plausibly believed that incongruent counterparts are grounded in absolute space, because (i) the difference between incongruent counterparts is a real difference

between bodies which requires a real ground, and because (ii) absolute space is the only means to differentiate the determinate regions in space that incongruent counterparts occupy.

(i) Before getting to the lone hand example, Kant first explains the *possibility* of incongruent counterparts to then assert that we have an ordinary actual experience of such objects: 'the shape of a body may be perfectly similar to the shape of the other, and the magnitudes of their extensions may be exactly equal, and yet there may remain an inner difference between the two' (2:382). We are in a good position to understand the relevance of this claim. I argued in the last section that a determinate difference in a real body requires a real ground, that is something really distinct from it. So, given something actual, we require a real ground for its existence.

(ii) Now why would absolute space be that in virtue of which we can explain the difference between left and right? The handedness property of bodies is determinately either right or left. The fact that there is such a determinate property, combined with the principle of determinate ground, justifies seeking a real ground. Whether or not a putative ground is genuine is tested counterfactually: if the ground does not exist, the determinate difference would not exist, and bodies would not be *left* or *right*. If that is the case, we have found the ground of handedness. Kant explicitly states that absolute space is the ground of the complete determination of bodies, because it is the ground of handedness.[22]

> 'What we are trying to demonstrate then is the following claim: The ground of the complete determination of a *corporeal form* does not depend simply on the relation and position of its parts to each other; it also depends on the *reference of that physical form to universal absolute space,* as it is conceived by the geometers' (my emphasis, 2:381).

Kant argues by giving an analysis of the concept of a "direction" in space to generate the view that there is a ground that determines the difference between left and right bodies. He writes that 'the ultimate ground, [is] the basis of which we form our concept of directions in space' (2: 379). Directions, such as left and right, in front and behind, above and below are patent in the way

they make a difference in everyday experience. If one mirrored writing on paper it would cease to be recognizable. A compass is not readable, and in fact useless, if I cannot distinguish North from South and East from West. Kant then argues that 'The distinctive characteristic in question consists in the *particular direction in which the order of the parts is turned*' (my emphasis, 2:380).

A direction here explains how to specify the feature that is not preserved when we mirror an object such that it can fit its counterpart. You are on the telephone with your friend and trying to explain to them exactly how to draw a triangle with a specific orientation. You tell them 'Start at point A. Draw a baseline of five centimetres, by moving your pen to the right of A. At the end of the baseline is now point B. Turn sharp left with a 90 degree angle and keep drawing for five more centimetres until you arrive at point C. Then turn sharp left again in a 45 degree angle for about 7 centimetres." By using directions such as *left* and *right*, your friend has now constructed a triangle that has a specific orientation—its parts are "turned" in some direction.

Now Kant carefully distinguishes the notion of a position from that of a direction to show that there is something contained in these directions that presupposes absolute space. He claims that "determinations of space are not consequences of the positions of the parts of matter relative to each other. On the contrary, the latter are the consequences of the former' (2:383). Kant means here that, generally, absolute space determines the position of parts of matter relative to each other, rather than space being itself the position of parts of matter relative to each other, because position sometimes involves being *turned* in a direction. By showing that parts of matter are *turned* in some direction, Kant believes to be able to show that they are in absolute space rather than space being a concept derivative from the position of parts of matter.

What is the difference between position and direction, then? The analysis aims to show that 'direction does not consist in the reference of one thing in space to another—that is really the concept of position—but in the relation of the system of these positions to the absolute space of the universe' (2:377). Kant's analysis is now clarifying how the position of objects in space differs from the concept of a direction in that there is something involved in the direction that resists

description by relation alone. If I tell my friend to draw the triangle without mentioning when to take a left turn, the position of A, B, and C to one another would be the same. But my friend and I could follow the exact same instructions and draw triangles which are mirror images of one another.

Directions can only be understood with reference to absolute space because things oriented in one direction rather than another occupy a different part of absolute space. Directions refer 'not to places in space' but 'rather to universal space as a unity, of which every extension must be regarded as a part' (2:378). Bodies in space occupy different parts of absolute space. A handed body occupies a region of space differently than its mirror image. One hand simply cannot fit into the part of absolute space occupied by its counterpart. Because Kant has shown that directions are distinct from mere positions, and because he believes that directions only make sense as references to absolute space, he has shown that absolute space is *really distinct* from the relations between objects. And recall that a real ground must also be really distinct from the thing it grounds.

We can further illuminate how absolute space differentiates objects in space by the space they occupy if we consider Newton's definition of absolute space in the *Principia.* Absolute space is characterized here as immovable (*immobilis*), and the order of parts of space is therefore also immovable. Let's say one hand occupies a certain part of space. If space were merely the movable order of objects, I could find a motion that maps one hand onto the other. If we cannot find a way to move a left-handed object through space such that it fits into the space of its counterpart, then, we have found a way in which objects occupy a truly different part of space. Thus, if absolute space were removed as a framework of orientation, the determinate difference between objects would not occur. This is the interpretation of Kant's argument that I offer.

A few final remarks. My account further provides a few indicators why profound systematic changes in Kant's philosophy may have occurred. The consequences of his paper may have been

graver than Kant anticipated in 1768. If my interpretation is correct, his argument for absolute space stands in tension with his prior relational theory of space. More precisely, the theory of space that Kant endorsed in 1768 *overdetermines* the shape of bodies. This is troubling. For one, even if the joint grounding of certain properties is not problematic, it is problematic that Kant endorsed two *competing* theories of what determines the shape of a body. If the intrinsic forces of substance and absolute space jointly ground the shape of bodies, we would be owed an account of their cooperation and co-determination. There are good reasons to think that this problem influenced Kant in the short time between the 1768 *Directions in Space* and the 1770 *Inaugural Dissertation.* If his previous theory of relations in space found a new ultimate ground in absolute space, he might have had a reason to rethink the matter altogether. It is no surprise then that Kant dispersed with the complex and perhaps unverifiable claim that space is an independent substance to assign it a place where it can be just as powerful in structuring spatial objects without competing with other fundamentals in the world, namely, the mind.

References

Allais, L. (2015) *Manifest reality: Kant's idealism and his realism*. Oxford: Oxford University Press.

Beck, L. W. (1969) *Early German philosophy: Kant and his predecessors.* Cambridge, M.A.: Harvard University Press.

Chalmers, D. (2019) chapters 3 Puzzles about spatial experience. In A. Pautz & D. Stoljar (Eds.), *Blockheads!: Essays on Ned Block's philosophy of mind and consciousness* (pp. 109–138). Cambridge, M.A.: MIT Press.

Couturat, L. (1905) *Les principes des mathématiques, avec un appendice sur la philosophie des mathématiques de Kant*. Hildesheim: Georg Olms Verlag.

Earman, J. (1971) Kant, incongruous counterparts, and the nature of space and space-time. reprinted in *Ratio* J. Van Cleve & R. E. Frederick (Eds.) *The Philosophy of Right and Left. Incongruent Counterparts and the Nature of Space*. Dodrecht: Kluwer Academic Publishers.

Earman, J. (1991) "On the Other Hand…: A Reconsideration of Kant. Incongruent counterparts, and absolute space. In J. van Cleve & R. E. Frederick (Eds.), *Philosophy of left and right: Incongruent counterparts and the nature of space*, *238*.

Falkenburg, B. (2020) *Kant's cosmology: From the precritical system to the antinomy of pure reason*. Berlin: Springer.

Friedman, M. (1985) 'Kant's theory of geometry'. *Philosophical Review*, 94, 455-506.

Gardner, M. (1969) *The new ambidextrous universe: Symmetry and asymmetry from mirror reflections to superstrings*. Mentor Books. *The Ambidextrous Universe*. Mineola: Dover Publications.

Hoefer, C. (2000) 'Kant's hands and Earman's pions: Chirality arguments for substantival space'. *International Studies in the Philosophy of Science*, 14, 237-56.

Hogan, D. (2009a) 'How to know unknowable things in themselves'. *Noûs*, 43, 49-63.

Hogan, D. (2009b). chapters 3 Kinds of rationalism and the non-spatiality of things in themselves. *Journal of the History of Philosophy*, 47, 355-82.

Hogan, D. (2020) Kant and the character of mathematical inference. In *Kant's philosophy of mathematics*, *I: The Critical Philosophy and Its Roots,* edited by Carl Posy and Ofra Rechter (pp. 125–154). Cambridge: Cambridge University Press.

Hogan, D. (2021) 'Handedness, idealism, and freedom'. *Philosophical Review*, 130, 385–449.

Kant, I. (1929) *Immanuel Kant's critique of pure reason* N. K. Smith (Translated by). London: Macmillan.

Kant. (1902) *Kants gesammelte Schriften*. Edited by the Berlin-Brandenburg (formerly: Royal Prussian) Academy of Sciences, the German Academy of Sciences in Berlin, and the Göttingen Academy of Sciences, 24 vols. Berlin: Walter de Gruyter.

Kant. (1992) *Theoretical philosophy, 1775–1770. Edited and translated by David Walford and Ralf Meerbote*. Cambridge: Cambridge University Press.

Kant. (1998) Kritik der reinen Vernunft. Nach der. *Original Ausgabe* J. Timmerman (Ed.), *1. And 2*. Hamburg: Felix Meiner.

Kant. Critique of pure reason. *Edited and translated by Paul Guyer and Allen Wood*. Cambridge: Cambridge University Press. (1998).

Langton, R. (1996) *Kantian humility: Our ignorance of things in themselves*. Oxford: Clarendon Press.

Langton, R. (2006) 'Kant's phenomena: Extrinsic or relational properties? A reply to Allais 1'. *Philosophy and Phenomenological Research*, 73, 170-85.

Lee, G., & Yelcin, S. (June 25–27, 2015). Finding meaning in a disoriented world Paper presented at the Finding Space: Space, and Consciousness Project Workshop Conference. Time. GIAS-NYU.

Leibniz, G. W., & Clarke, S. (2000) *Correspondence* R. Ariew (Ed.). Indianapolis: Hackett Publishing Company.

Maudlin, T. (1993) 'Buckets of water and waves of space: Why spacetime is probably a substance'. *Philosophy of Science*, 60, 183–203.

Mühlhöltzer, F. (1992) Das Phänomen der inkongruenten Gegenstücke aus Kantischer und heutiger Sicht. *Kant-Studien*, *83*(4), 443f.

Nerlich, G. (1973) 'Hands, knees, and absolute space'. *Journal of Philosophy*, 70, 337-51.

Newton, I. (1999) *The principia: Mathematical principles of natural philosophy* I. B. Cohen & A. Whitman (Translated by). Oakland: University of California Press.

Pooley, O. (2003) Handedness, parity violation, and the reality of space. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 250–280). Cambridge University Press.

Remnant, P. (1963) Incongruent counterparts and absolute space. *Mind*, *72*(287), 393–399.

Rukgaber, M. S. (2016) 'The asymmetry of space: Kant's theory of absolute space in 1768'. *Kantian Review*, 21, 415-35.

Rusnock, P., & George, R. (1995) 'A last shot at Kant and incongruent counterparts'. *Kant-Studien*, 86, 271.

Sklar, L. (1974a) 'Incongruous counterparts, intrinsic features, and the Substantiviality of space'. *Journal of Philosophy*, 71, 277-90.

Sklar, L. (1974b) *Space, time and spacetime*. Oakland: University of California Press.

Stang, N. F. (2012) 'Kant on complete determination and infinite judgement'. *British Journal for the History of Philosophy*, 20, 1117-139.

Stang, N. F. (2016) *Kant's modal metaphysics*. Oxford: Oxford University Press.

Stang, N. F. (2019) A guide to ground in Kant's lectures on metaphysics. In C. D. Fugate (Ed.), *Kant's lectures on metaphysics: A critical guide* (pp. 74–101). Cambridge: Cambridge University Press.

Van Cleve, J. (1987) 'Right, left, and the fourth dimension'. *Philosophical Review*, 96, 33-68.

Wittgenstein, L. (1974) *Tractatus logico-philosophicus*. Milton Park: Routledge.

Notes

1 My interpretation does not reflect Kant's mature position. Thirteen years after *Directions in Space* was published Kant extensively argues in the *Critique of Pure Reason* that space is neither relational nor absolute. It is mind-dependent. In Kantian phrase: space is the *a priori* form of how we receive objects in intuition. That space is the *a priori* form of our intuition is - according to Lucy Allais - the central claim of Kant's transcendental idealism;. Not everyone agrees with this view. Unfortunately, it is impossible to list the many compelling views on this issue in a single footnote. In lieu of that, I will cite those works that most shape my understanding of transcendental idealism..

2 A mesmerizing array of philosophers engage with the argument. References were made by people as diverse as Ludwig Wittgenstein in Tractatus Logico-Philosophicus, and Gilles Deleuze in Difference and Repetition. But apart from engaging with the example, a surprising number of contemporary philosophers have sat to examine Kant's text. For instance, a recent study by two prominent contemporary formal semanticists Geoffrey Lee and Seth Yelcin & David Chalmers. Apart from these discussions in philosophy of language and in philosophy of mind, most attention came from philosophers of physics among them.

3 Both Remnant and Van Cleve hedge their opposition to indeterminacy. According to Remnant, indeterminacy would be an option if we can rule out that incongruent counterparts are handed by their own nature, while Van Cleve believes that indeterminacy follows from accepting certain facts about the fourth spatial dimension.

4 Lewis White Beck notes for instance that some of the formulations in *Directiosn in Space* make no sense at all from the vantage point of the critical approach in *Early German Philosophy: Kant and his Predecessors*. White Beck is a great example of an interpretation that points out how radical Kant's views on the ontology of space change (I touch on this in section 3 of this paper), what I show is that this is a result

of some continuously held views on correct metaphysical reasoning which only really changed with Kant's change in views in the critical turn.

5 This view has partially been explored by Desmond Hogan in *Handedness, Idealism, and Freedom* and James Van Cleve in his *Right, Left and the Fourth Dimension*, but I come to a different conclusion than Van Cleve does as will later be shown.

6 I will cite most of Kant's works by indicating the volume and page number in which they appear in the *Academy edition* of Kant's collected works, so in this case page 382 of the second volume (4:285). As is customary, I cite the *Critique of Pure Reason* by indicating the page number of both the first (A) and second (B) edition where available (e.g. (A23/B37)).

7 This theme also runs through the Leibniz-Clarke correspondence. Certain facts about space and time are explained as depending on a prior entity. There must be a sufficient reason why God created the universe in this particular location, rather than another one. In other words, the spatial location of the universe depends on a Divine reason. Some interpreters like Lewis White Beck for instance read Kant's argument as a response to the Leibniz-Clarke Correspondence. Particularly Leibniz' third letter in the Leibniz-Clarke correspondence provides a useful background to themes discussed here, but I omit a discussion of their correspondence because the interpretive issues I raise are not alleviated by providing an interpretation of the points Leibniz raises. White Beck, *Early German Philosophy,* 449 f.

8 Extrinsically does not simply follow from dependence, since some (secondary) properties of an object can depend on other (primary) properties. Kant believed that "having a color" depended on "being extended." So, dependence and extrinsically can come apart. The same holds for intrinsicality, which I do not touch on in this shorter version of the paper.

9 John Earman and Felix Mühlhöltzer respond to Remnant's criticism by pointing to the fact that in our universe mirror images of objects do not maintain the same properties, that is parity is not conserved. John Earman, "Kant, Incongruous Counterparts, and the Nature of Space and Space-Time." Although it is unclear to me why the non-conservation of parity would render Kant's reasoning any more consistent at this point.

10 A refreshing contrast to this view is provided by Graham Nerlich who moved to defend the dependence of handedness on general features of space by invoking higher dimensional spaces. The fact that we can superimpose two objects which are incongruent in the n^{th} dimension, but render them congruent in n+1 dimensional space points to a dependence of spatial orientation on general features of space.

11 Remnant brings this point up against Kant, too.

12 This confusing assertion led Carl Hoefer to ignore "the cryptic line about an 'inner ground', which I have never seen plausibly interpreted."

13 The standard English translation in Walford and Meerbote's edition omits this phrasing. In the German the passage reads, "man mag ihn drehen und wenden wie man will."

14 His views on mathematics changed after the critical turn, specifically Kant's sophisticated theory of metaphysical truth (as in the validity of synthetic a priori judgments) forces on him a finer grained account of what counts as a mathematical object and mathematical inference. In mathematics constructions in pure intuition "express the [mathematical] concept" (A714/B742), whereas in metaphysical reasoning concepts have to refer to intuition. Whether this means that Kant requires extra logical means for his notion of mathematical inference (and whether this is acceptable or not) is a matter of dispute.

15 The suggestion that incongruent counterparts could not be mathematically constructed is egregious in its own right. Kant even describes constructing incongruent counterparts using a compass and a ruler in the middle of a section on mathematics in the *Prolegomena*. The point is not that the tools for construction described by Euclid are sufficient to construct any mathematical object. Kant emphasizes time and again that a continuous, homogenous space must be *presupposed* for these constructions to have the content they have. Mathematical constructions can only work *in* pure space, not independent of them, and any suggestion that he believed they could do so misconstrues his profound remarks on the continuity, infinity, and homogeneity of space.

16 Brigitte Falkenburg offers another way out of this question by suggesting that *because* mathematics failed to explain incongruent counterparts, Kant is *forced* into a metaphysical explanation. I find this claim stretches the textual evidence available and is not philosophically compelling. Nowhere does Kant articulate the view that what cannot be explained by mathematics must be explained metaphysically, nor does it seem to be a reasonable belief to hold.

[17] Prima facie, the two readings are not mutually exclusive. Even when we have found an account that determines how to construct a geometric object, we might still wonder why this is so. But a simple way to hold these questions apart is to ask: how do relations fail in leaving something indeterminate? The difference comes down to this: the mathematical reading focuses on questions about construction. How is an object constructed such that it is handed? If we can find an internal relation among parts that constructs an object such that it is handed, the investigation is complete. The metaphysical proposal asks why are things handed? What determines the property? The answers overlap but the method of argument differs.

[18] Van Cleve is not the only person to argue for a metaphysical reading. Desmond Hogan for example bases much of his argumentative strategy on this metaphysical claim.

[19] This is one of the central tenets of Kant's pre-critical dynamics and is a viable alternative understanding of how to interpret what internal properties *other* than construction Kant could have in mind. F.

[20] Although most scholars of Kant agree that he no longer believes that we have any knowledge whether things in themselves are indivisible, the extent to which they are endowed with forces whose effects we can have knowledge of has been the topic of debate between Lucy Allais and Rae Langton, with Allais arguing convincingly that the textual evidence and his foundational beliefs about the status of metaphysics better support the view that Kant thought that things in themselves ground appearances in just the way Langtonian irreducibility would rule out decisively.

[21] As the title indicates, the task of the *New Elucidation* is to provide 'absolutely certain first principles of metaphysical cognition.' As such, the paper aims to reexamine principles of metaphysical cognition in use at the time. What is a principle of metaphysical cognition? A principle of metaphysical cognition tells you how to think about reality. The principle of noncontradiction, for instance, tells you that a description of reality cannot contain contradictions. If I were to think about my friend Ish sitting across the table from me, and I were to say that Ish is simultaneously moving their pen and not moving their pen, I would be contradicting myself. That is not how Ish is, that is not we think about reality. Maybe I am dreaming and my thought has become incoherent etc.

[22] As an aside, Kant notes that there are some difficulties in showing this relation between absolute space and handed bodies, because absolute space is not perceptible. We can identify the difference between left- and right-oriented bodies easily. These differences "which exist between the bodies and which depend exclusively on this ground alone, can be immediately perceived" (2:381), so we have good data for a real difference between bodies. However, the reference of bodies to absolute space itself cannot be perceived. "This relation to absolute space, however, cannot be immediately perceived." (2:381) Why would Kant make note of this? Compare this case to another real grounding relation, for instance, the causal interaction of balls in a game of pool. If I forcefully push the cue onto the ball, the ball will move. I can perceive the movement of the ball as a real change in motion and trace it back to the motion of the cue on it, its real ground, which is also perceivable. The connection between space and handed bodies cannot be established in this way because absolute space itself is not perceivable. So, the real grounding relation cannot be shown by way of perceptible changes. Much like Kant's argument that God safeguards the interaction of substances, the argument cannot be established by way of direct empirical evidence.